Final Exam

Mathematical Methods of Bioengineering
Ingeniería Biomédica - INGLÉS

28 of May 2021

Please write neatly. Answers which are illegible for the grader cannot be given credit. Question 3 d) is optional. You have 180 minutes time to complete your work. You are allowed to use a calculator and two sheets with annotations.

Problems

1. Consider the plane curve given by the parametric equation $\mathbf{c}(t) = (e^{-t}\cos t, e^{-t}\sin t),\ t \in \mathbb{R}$.

   a) (**0.5 points**) Find the tangent line at the point $(-e^{\pi}, 0)$.

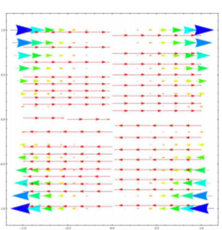
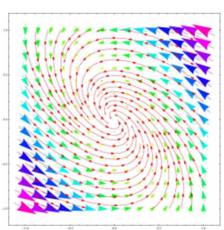
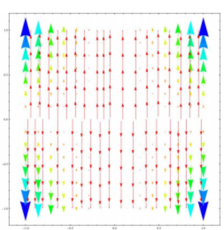
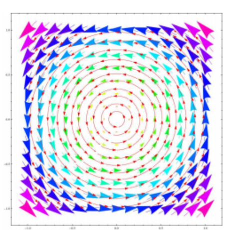
   b) (**0.75 points**) Find the length of curve between the points $(-e^{\pi}, 0)$ and $(1, 0)$. Find now, the length of the curve as t varies in $[0, \infty)$. Which curve is longer?

   c) (**0.75 points**) Find the tangent unit vector. Reparametrize the path in terms of the arclength parameter s[1]. What is the speed determined by this new parametrization?

   d) (**1 point**) Find $\kappa(t)$, the curvature of the path. Find when the curvature is equal to the curvature of the unit circle, and when is minimum and maximum.

2. a) (**0.6 points**) The pictures display flow lines of vector fields in two dimensions. Match them and explain your choice.

Field	Enter 1-4
$\vec{F}(x,y) = \langle 0, x^2y \rangle$	
$\vec{F}(x,y) = \langle x^2y, 0 \rangle$	
$\vec{F}(x,y) = \langle -y-x, x \rangle$	
$\vec{F}(x,y) = \langle -y, x \rangle$	


1

2

3

4

   b) (**1 point**) Calculate the flow line $\mathbf{x}(t)$ of the vector field $\mathbf{F}(x,y,z) = (1, -3y, z^3)$ that passes through the point (3,5,7) at $t = 0$.

3. You invent a **3D printing process** in which organic tissues of variable density can be printed. To try this out, we take a triangular tissue E, with vertices $A = (1,2)$, $B = (3,2)$ and $C = (3,4)$ in centimetres, which has density $f(x,y) = 24x$ g/cm^2.

   a) (**0.25 points**) Describe the tissue E as a type I region.

[1] When computing the arclength parameter you can choose any "base point", for example $\mathbf{c}(0)$.

b) (**0.3 points**) Describe the tissue E as a type II region. Is E a type III region?

c) (**1.1 point**) Find the total mass of the tissue.

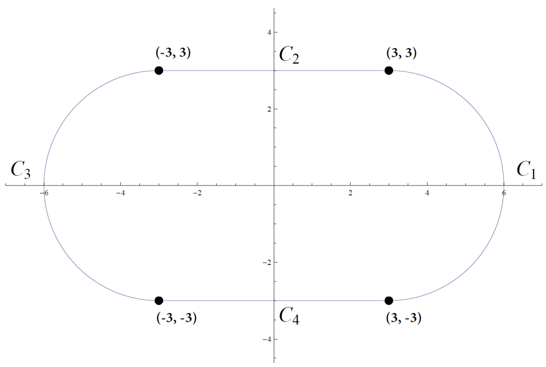
d) **(+0.5)** Evaluate the integral in (c) by making the substitution $x = u, \ y = u + v$.

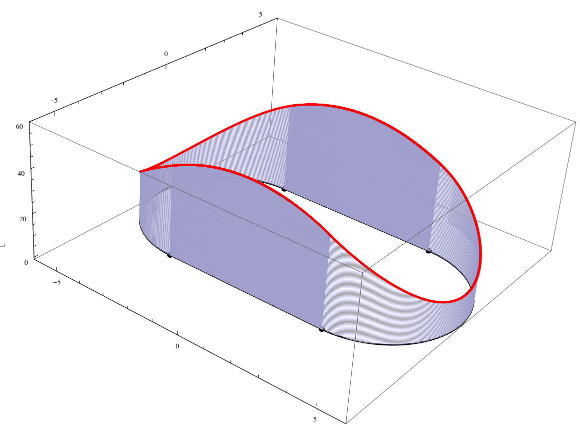
4. Consider a virtual reality glasses case, with the shape shown in the figure below. The **VR glasses** form part of a medical device training platform focused on improving surgical skill with *Virtual Training*.

   a) (**1 points**) The base of the case lies in the xy-plane as in the figure below on the left. It is modelled as: left and right boundary semicircumferences (C_3, C_1) and top and bottom are straight segments (C_2, C_4). Parametrize the case base[2].

   b) (**1.5 points**) Compute the area of the virtual reality *case* if the height of the glasses lying in the xy-plane is given by the function $f(x, y) = 50 - x^2$ (centimeters).

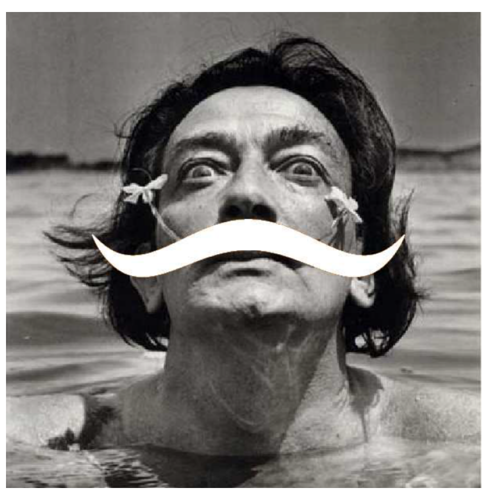
   Note: $\cos^2(t) = \frac{1+\cos 2t}{2}$.





5. (**1.5 points**) Compute the area of the **moustache region** (see illustration below) which is enclosed by the curve:

$$\mathbf{r}(t) = (5\cos t, \sin t + \cos 4t), \quad 0 \le t \le 2\pi.$$



Note: $\sin\alpha\cos\beta = \frac{1}{2}\big(\sin(\alpha - \beta) + \sin(\alpha + \beta)\big)$.

[2] You may use that a parametric equation of a circle of radius r centred at a point (x_0, y_0) is: $\mathbf{c}(t) = (x_0 + r\cos t, y_0 + r\sin t)$, $t \in (0, 2\pi]$.